NCC-003-016202 Seat No. ________

M. Sc. (Mathematics) (CBCS) (Sem. II) Examination

April / May – 2017

Mathematics : CMT - 2002

(Complex Analysis) (Old Course)

Faculty Code : 003

Subject Code : 016202

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all questions.

(2) Each question carries 14 marks.

1 Answer any seven questions : **2×7=14**

(i) Define the chrodal metric on $\mathbb{C}_\infty$.

(ii) If $z_2, z_3, z_4 \in \mathbb{C}$ are distinct then (z, z_2, z_3, ∞) = ________ and (z, z_2, ∞, z_4) = ________.

(iii) True or false ? Justify. S defined by $S_z = \overline{z}$ is a bilinear transformation.

(iv) If $\lambda \in \mathbb{C}, \lambda \neq 0, \lambda \neq 1$ then find the fixed points of the bilinear transformation S defined by $S_z = \lambda z$.

(v) If $z, w \in \mathbb{C}$ and $\gamma : [0,1] \to \mathbb{C}$ is defined by $\gamma(t) = (1-t)z + tw$ $\forall\, t \in [0,1]$ then find $V(\gamma)$.

(vi) If $G \subset \mathbb{C}$ is a region, $f : G \to \mathbb{C}$ is analytic and $|f| : G \to \mathbb{R}$ is a constant function then prove that $f : G \to \mathbb{C}$ is a constant function.

(vii) 0 is ________ of $e^{\frac{1}{z}}$

(a) removable singularity

(b) a simple pole

(c) an essential singularity

(d) not a singularity

(viii) 0 is a removable singularity of ________.

(a) $\cos\left(e^{\frac{1}{z}}\right)$ (b) $\cos z$

(c) $\frac{\sin z}{z}$ (d) $\frac{1}{z}$

(ix) Find the right side of the x-axis w.r.t. the orientation $(-1, 0, 1)$.

(x) If $f : D \to D$ is analytic and $f(0) = 0$ then ________.

(a) $\left|f\left(\frac{3}{4}\right)\right| \le \frac{3}{4}$

(b) $f(z) = cz$, for none $C \in ¢$, $|C| = 1$

(c) $|f'(0)| = 2$

(d) $\left|f\left(\frac{1}{2}\right)\right| > \frac{1}{2}$

2 Answer any two questions : **2×7=14**

(a) Given two circle Γ_1, Γ_2 in $¢_\infty$ and distinct $z_2, z_3, z_4 \in \Gamma_1$, distinct $w_2, w_3, w_4 \in \Gamma_2$ prove that ∍ a unique bilinear transformation S rt $S(\Gamma) = \Gamma'$ and $S(z_j) = w_j, \forall\, j = 2, 3, 4$

(b) Define the right side, left side of a circle Γ in $\mathbb{C}_\infty$ w.r.t. an orientation of Γ. Find the right side of the imaginary. axis L w.r.t. the orientation $(-i, 0, i)$ and the left side of the unit circle Γ with centre at o w.r.t. the orientation $(-i, -1, i)$.

(c) Find the bilinear transformation S taking $1 \to 0, 0 \to \infty, \infty \to 1$.

3 (a) If γ is a rectifiable curve in $\mathbb{C}$, $f = \{\gamma\} \to \mathbb{C}$ is continuous and $C \in \mathbb{C}$ then prove that 7

$$\int_\gamma f(z)dz = \int_{\gamma+c} f(z-c)dz.$$

(b) State, without proof, the fundamental theorem of calculus for line integrals. If $f : G \overset{\text{open}}{\subset} \mathbb{C} \to \mathbb{C}$ is continues with primitive and $\gamma, \sigma : [a,b] \to G$ are rectifiable s.t. $\gamma(a) = \sigma(a), \gamma(b) = \sigma(a)$ then prove that $\int_\gamma f = \int_\sigma f$. 7

OR

(c) Prove that $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z} ds = 2\pi, \ \forall \ z \in \mathbb{C}, |z| < 1$.

(d) State, without proof, Cauchy's integral formula for higher derivatives of an analytic function $f : G \to \mathbb{C}$.

Evaluate $\int_\gamma \frac{\sin z}{z^3} dz$, where $\gamma(t) = \ell^{it}$, $\forall t \in [0, 2\pi]$

4 Answer any two questions : **2×7=14**

(a) State and prove minimum modulus theorem.

(b) Define removable singularity of complex function "f" of a complex variable and given an example. Prove that $a \in ¢$ is a removable singularity of f iff $a_n = 0, \; \forall \, n \le -1$ in the Laurent's expansion $\sum_{n=-\infty}^{\infty} a_n (z-a)^n$ of f at a.

(c) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$.

5 Answer any two questions : **2×7=14**

(a) State, without proof, Rouche's theorem. Prove that $3z^7 + 5z - 1$ has exactly one zero in $|z| < 1$ and is a real zero in $(0, 1)$.

(b) Find the Laurent's expansion of $f(z) = \frac{z+2}{z^2 - 2z - 3}$ in

(i) $|z| < 1$

(ii) $|z| > 3$

(c) Let $f : G \overset{\text{open}}{\subset} ¢ \to ¢$ be analytic and one-one. Then prove that $f'(3) \ne 0, \forall \; 3 \in G$

(d) Find $\int_{\gamma} \frac{1}{z} dz$, where $\gamma = [1-i, 1+i, -1+i, -1-i, 1-i]$